Phase transitions for a collective coordinate coupled to Luttinger liquids

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We study various realizations of collective coordinates, e.g. the position of a particle, the charge of a Coulomb box or the phase of a Bose or a superconducting condensate, coupled to Luttinger liquids (LL) with N flavors. We find that for Luttinger parameter $\frac{1}{2} < K < 1$ there is a phase transition from a delocalized phase into a phase with a periodic potential at strong coupling. In the delocalized phase the dynamics is dominated by an effective mass, i.e. diffusive in imaginary time, while on the transition line it becomes dissipative. At $K = \frac{1}{2}$ there is an additional transition into a localized phase with no diffusion at zero temperature.

Diffusion and propagation of massive particles surrounded by a bath is one of very challenging problem of condensed matter. Historically it started with the celebrated Brownian motion [1] in which the interactions between a classical particle and the microscopic motion of the classical bath, lead to a diffusion, connected by the Einstein relation to a finite friction.

This problem gets incredibly more complicated when the bath becomes quantum. Indeed the excitations of the bath can lead, by Anderson orthogonality effects to a modification of the motion of the quantum particle or the collective coordinate coupled to the bath [2]. One of the realization of such problem is the polaron problem [3] where the interaction with the vibrations of the lattice leads to an increase of the mass of the particle and even potentially to self trapping. This type of problem has recently benefitted from the recent progress in cold atomic systems [4]. Indeed in such systems impurities in quantum baths can be realized in a variety of manners ranging from fermi- or bose- mixtures to ions in condensates, and at various dimensionalities [5–14].

A situation of special interest is provided by a one-dimensional bath for which the bath-bath correlations can become highly non-universal due to the existence of the powerlaw correlations characteristics of a Luttinger liquid (LL) [15]. In that case special effects can potentially occur, as clear from the static impurity case [16] and mobile ones coupled to single baths [17, 18]. In particular it was shown recently [19] that this led to a new universality class for the motion of the impurity, for which in particular subdiffusion can occur. This very rich situation was explored further. On the theory side both diffusive [20–22], kicked [23, 24] and driven impurities [25–27] were considered. On the experimental side driven impurities [12], mixtures of ^{87}Rb and ^{41}K [13] and ^{87}Rb experiments with local addressability [14] were successful implementation of the one dimensional problem.

In this paper we study the physics of a collective coordinate coupled to N Luttinger baths ($N \gg 1$), e.g. a particle position, charge of a Coulomb box, a phase of a Bose-Einstein condensate (BEC), or a phase of a superconducting grain. For concreteness, the presentation uses the particle coordinate language while other realizations are discussed below. We show that this problem exhibits a novel localization-delocalization transition as a function of the bath characteristics, as summarized in Fig. 1. The localized and delocalized states are separated by a line on which the motion is simply diffusive. We discuss the consequences for the linear response of the impurity to an external force, and propose potential experimental realizations of this device in cold atomic systems and condensed matter ones.

We consider a particle of mass $\tilde{M}$ coupled to a LL with a contact interaction $\tilde{H}_{\rm int} = g\rho(\tilde{X})$ where $\tilde{X}$ is the operator measuring the impurity position and $\rho(x)$ the density is the LL. We study this model in the large N limit, so the impurity becomes coupled to N independent LL and the interaction becomes $H_{\rm int} = g\sum_{i=1}^{N}\rho_i(\tilde{X})$. The action of the system can be computed by a cumulant expansion in powers of g and only the second order cumulant remains when $g^2N = O(1)$. Indeed the fourth order cumulant is of order $g^4N \sim 1/N$ and can be neglected. Using the expression of the density in a LL [15] $\rho(x,\tau) = \rho_0 - \frac{1}{\pi}\partial_x\phi(x,\tau) + \rho_0[\mathrm{e}^{2i(\pi\rho_0 x - \phi(x,\tau))} + h.c.]$ where $\phi(x,\tau)$ is the bosonic phase, and performing the Gaussian integration over the LL Hamiltonian, the action becomes

$$S_{\rm eff} = \frac{M}{2}\int_\tau (\dot{X})^2_\tau - \frac{\eta\Lambda^2}{2\pi}\int_\tau\int_{\tau'}\frac{\cos(X_\tau - X_{\tau'})}{(\Lambda(\tau-\tau'))^{2K}} \qquad (1)$$

where we have used the dimensionless variables $X = 2\pi\rho_0\tilde{X}$ and $M = \tilde{M}/(2\pi\rho_0)^2$, $\eta = 2\pi g^2\rho_0^2 N/\Lambda^2$, τ is the imaginary time, u the velocity of excitation in the LL, and K the Luttinger parameter that controls the power-law decay of the correlation functions. A frequency cutoff

$\Lambda = u/\alpha$ is used to have a dimensionless coupling η where $\alpha \approx 1/\rho_0$ is the natural momentum cutoff of the LL. In the above expression only the oscillating (backscattering) term in the density has been retained. Indeed the $\partial_x \phi(x,\tau)$ interaction can be integrated, leading at long times to $(X_\tau - X_{\tau'})^2/(\tau-\tau')^4$, i.e. an ω^3 term in frequency which can be neglected relative to the bare kinetic energy term of the impurity $M\omega^2$. We have used that for a LL one obtains [15] $\langle \mathrm{e}^{i\phi(X_\tau,\tau)-i\phi(X_{\tau'},\tau')}\rangle_{LL} \sim [(X(\tau)-X(\tau'))^2 + u^2(\tau-\tau')^2]^{-K}$. We have also made the additional assumption, which will be verified in what follows that the impurity is less than ballistic and thus that $\langle (X_\tau - X_{\tau'})^2\rangle \ll u^2(\tau-\tau')^2$.

To solve for the thermodynamics of (1) we consider first a renormalization group (RG) process [28] valid for large η, which was also applied to the $K=1$ case [29]. The action (1) is approximated by its short time form where it becomes Gaussian

$$S_0 = \frac{1}{2}\int_\omega [M\omega^2 + \eta C_K \Lambda^{2-2K}|\omega|^{2K-1}]|X(\omega)|^2 \qquad (2)$$

where $\int_\tau \frac{1-\cos(\omega\tau)}{\tau^{2K}} = -2\Gamma(1-2K)\sin(K\pi)|\omega|^{2K-1} \equiv \pi C_K|\omega|^{2K-1}$ so that $C_K = 1 - 0.85(K-1) + O(K-1)^2$. The cutoff Λ is replaced by Λ' and the interaction is averaged with S_0 in the small frequency interval $\Lambda' < \omega < \Lambda$ leading to $d\Lambda/\pi C_K \eta\Lambda$. The action has then a renormalized coefficient $\eta^R(\Lambda')^{2-2K}$ where

$$\eta^R = \eta\left\{1 + [(2-2K) - \frac{1}{\pi\eta}]\ln\frac{\Lambda}{\Lambda'}\right\} \qquad (3)$$

with $C_K \to 1$ to 1st order in either $1/\eta$ or $1-K$. Hence if $K \geq 1$ η^R flows to small values, while if $K<1$ there is an unstable fixed point at $\eta_c = \frac{1}{2\pi(1-K)}$. $\eta > \eta_c$ flows to large values, while $\eta < \eta_c$ flows to smaller values of η. One can integrate (3) when $\eta < \eta_c$ down to $\eta^R \approx 1$ below which the RG is not controlled. The new cutoff is interpreted as an effective mass [29, 30] M^*

$$\frac{1}{M^*} \approx \Lambda[1 - \pi\eta(2-2K)]^{\frac{1}{2-2K}} \qquad (4)$$

which for $\pi\eta(2-2K) \ll 1$ but $\pi\eta \gg 1$, i.e. far from the transition point, represents an exponentially large mass $M^* \sim \mathrm{e}^{\pi\eta}$ as for the $K=1$ case [29, 30].

To confirm this scenario, and study the properties of the three resulting phases, we follow a variational scheme [31] where we find the best quadratic action approximating the original action (1). The corresponding Green's function $1/f(\omega)$ is a solution of the self-consistent equation

$$f(\omega) = M\omega^2 + \frac{2}{\pi}\eta\Lambda^{2-2K}\int_0^\infty d\tau \frac{1-\cos\omega\tau}{\tau^{2K}}\,\mathrm{e}^{-\int_0^\Lambda \frac{1-\cos\omega'\tau}{\pi f(\omega')}} \qquad (5)$$

We note first that at $\omega \approx \Lambda$ the solution is $f(\omega) - M\omega^2 \sim |\omega|^{2K-1}$. In the following we focus on $\omega \ll \Lambda$ and on $K<1$. As a first option we consider $f(\omega) = \eta^* C_K \omega^{2K-1}/\Lambda^{2K-2}$. The integral in the exponent converges as $\tau \to \infty$, so it is $\int_0^\Lambda d\omega'/(\pi f(\omega')) = [\pi\eta^* C_K(2-2K)]^{-1}$, hence (5) reduces to

$$\eta^* = \eta\mathrm{e}^{-[\pi\eta^* C_K(2-2K)]^{-1}} \qquad (6)$$

This equation has solutions only if η is sufficiently large, i.e. $\pi C_K \eta(2-2K) > e$. A second possible solution is $f(\omega) = \eta^*|\omega|$. The exponent behaves as $\int_0^\Lambda \frac{1-\cos\omega\tau}{\pi\eta^*\omega} = \frac{1}{\pi\eta^*}\ln\Lambda\tau$, since the τ integral is dominated by long τ, hence

$$\eta^*\omega = \frac{2}{\pi}\eta\Lambda^{2-2K-1/\pi\eta^*}\int_0^\infty d\tau \frac{1-\cos\omega\tau}{\tau^{2K+1/\pi\eta^*}} = \eta\omega \qquad (7)$$

which is a consistent solution on a line $\frac{1}{\pi\eta} = 2(1-K)$. The third possible solution is similar to the bare one $f(\omega) = M^*\omega^2$, then the exponent behaves as $\int_0^\infty \frac{1-\cos\omega\tau}{\pi M^*\omega^2} = |\tau|/2M^*$, leading to

$$1 = \frac{M}{M^*} + \frac{\eta}{\pi}[M^*\Lambda]^{2-2K} \qquad (8)$$

where the τ integral is cutoff by M^* and then an expansion in ω is possible. This solution is acceptable only for intermediate or weak coupling and then $M^* \approx M$.

The existence of the three possible regimes in the variational approach shows that there is a connection between this method and the RG process [31]. One can replace (5) by

$$f'(\omega) = \tilde{B}(\eta)\left(\frac{\omega}{\Lambda}\right)^{2K-2}\mathrm{e}^{-\int_\omega^\Lambda \frac{d\omega_1}{\pi f(\omega_1)}} + \omega f''(\omega) \qquad (9)$$

where $\tilde{B}(\eta)$ can be found as a perturbative series. RG proceeds by defining scaling functions such that the frequency dependence is replaced by a renormalized $\bar{\eta}(\omega)$, i.e. $f(\omega) = \bar{\eta}(\omega)\omega$ with $\bar{\eta}(\Lambda) = \eta$, $f'(\omega) = B[\bar{\eta}(\omega)]$, $f''(\omega) = \frac{1}{\omega}C[\bar{\eta}(\omega)]$. The flow is given by

$$\frac{\partial\bar{\eta}(\omega)}{\partial\ln\omega}|_\Lambda = B(\eta) - \eta = \frac{C(\eta)}{B'(\eta)} \qquad (10)$$

i.e. $B(\eta) = \eta$ and $C(\eta) = 0$ are fixed point conditions. Eq. (9) and its derivative at $\omega = \Lambda$ yields

$$B'(\eta)[B(\eta)-\eta][-B'(\eta) + 2K + \frac{1}{\pi\eta}] = (2K - 2 + \frac{1}{\pi\eta})B(\eta) + B''(\eta)[B(\eta)-\eta]^2 \qquad (11)$$

with boundary conditions at a large η_0 from a perturbative analysis $B(\eta_0) = \eta_0(2K-1) + \frac{1}{\pi}$, $B'(\eta_0) = 2K-1$.

Note first that a fixed point $B(\eta) = \eta$ is achieved at $2K - 2 + \frac{1}{\pi\eta} = 0$ to all orders, consistent with Eq. (7), as an unstable fixed line. Note next that $f(\omega) \sim \omega^2$ is a solution with $B(\eta) = 2\eta$, consistent with Eq. (8). It

corresponds to a fixed point at $\bar{\eta}(\omega) \sim \omega \to 0$. Finally, for large η one can solve Eq. (5) perturbatively, leading to $f(\omega) = \eta^R(\omega)\omega$ with $\eta^R(\omega)$ given formally by Eq. (3) with the replacement $\Lambda' \to \omega$. Using the RG structure this can be integrated and for $\frac{1}{2\pi\eta} \ll 1 - K$ it yields $f(\omega) = \eta\Lambda^{2-2K}\omega^{2K-1}$, i.e. S_0 of Eq. (2) is a fixed point action for large η.

The analysis leads thus to three different possible behaviors for the system:
i) At $1 - K = \frac{1}{2\pi\eta}$ the system has a dissipative behavior with $f(\omega) = \eta|\omega|$.
ii) The case $1 - K < \frac{1}{2\pi\eta}$ flows to small η and eventually to an $M^*\omega^2$ form, with $\langle(X_\tau - X_0)^2\rangle \sim |\tau|$, which corresponds to a delocalized phase. The effective mass M^* is identified by the RG flow, as in Eq. (4). Note that even in this delocalized phase, some effects of the underlying quasi-long range periodicity of the LL with the wavevector $2\pi\rho_0$ are still felt by the particle. Indeed its correlation *at that periodicity* are only very slowly decaying $\langle\cos X_\tau \cos X_0\rangle \sim \tau^{-2K}$. This indicates that the particle has a much greater chance to be found at some particular places on the chain. This can be understood qualitatively by the argument that the particle moves in the "charge density wave" of wavevector $2\pi\rho_0$ provided by the LL, hence the particle diffuses predominantly by tunneling between lattice sites spaced by $1/\rho_0$. On the mathematical side, this property which is apparent in a first order calculation in η [31] is in fact known in general in the context of XY models with long-range interactions [32].
iii) The case $1-K > \frac{1}{2\pi\eta}$ flows to large η with eventually $f(\omega) \sim \omega^{2K-1}$. From this form one could naively expect that the correlations of $\langle[X_\tau - X_0]^2\rangle$ to be convergent and thus this phase to be a localized one. The situation is in fact more subtle and we discuss this phase in more details below.

A summary of the various regimes can be found in Fig. 1 and the corresponding correlation functions are indicated in Table I.

Let us now discuss in more details the physics of the phases, in order to ascertain that they are not mere artefact of the variational approach. A priori the variational scheme is reliable for large η, i.e. for the transition line near $K = 1$. For $K < 1$ one can complement the above analysis by a mean-field approach similar to the one used in the context of XY models with long-range interactions [33]. We take $h = \langle\cos X_\tau\rangle$ as an order parameter. The interaction term in (1) decouples as $\eta\Lambda^{2-2K} h \int_\tau \cos X_\tau \int_{\tau'} |\tau - \tau'|^{-2K} = \eta\Lambda h \frac{1}{2K-1} \int_\tau \cos X_\tau$. The self consistency relation, linear in h, is $1 = \eta\Lambda\frac{1}{2K-1}\int_{\tau'}\langle\cos_\tau \cos_{\tau'}\rangle_0 = 4\eta M\Lambda\frac{1}{2K-1}$; it yields the critical line $\eta_c = \frac{2K-1}{4M\Lambda}$ above which $\langle\cos X_\tau\rangle \neq 0$. We expect the mean field result to be more reliable near $K = \frac{1}{2}$, where the range of the interaction increases. As K increases from $K = \frac{1}{2}$ fluctuations will increase the

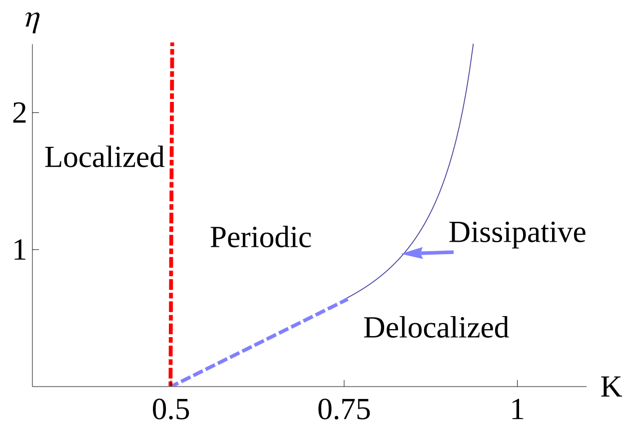

FIG. 1. Phase diagram for an impurity in a bath of LL as a function of the LL parameter K and the interaction parameter between the impurity and the bath η. Four regimes can occur (see text) in which the impurity is delocalized, just dissipative, periodically localized, or localized. Dashed lines indicates boundaries out of the control of the perturbative RG. The corresponding correlation functions are given in Table I.

critical value, eventually joining the transition line with the variational form $\eta_c = \frac{1}{2\pi(1-K)}$ near $K = 1$. Note that mean field exponents become valid [33] when $K < 3/4$, e.g. $\langle\cos X_\tau\rangle \sim \sqrt{\eta - \eta_c}$. In Fig. 1 we plot the transition line as an interpolation between the mean field at $K < 3/4$ and the variational form at $K > 3/4$. We see that the point $K = 1/2$ plays an important role, not captured by the variational or RG approaches. Below this point the interaction is so-long range that an ordered phase would exist, within the mean-field solution, for arbitrary strength of the coupling η.

We reconsider now the periodic phase. Since the ordered phase would have many degenerate minima of the order parameter, we expect the presence of instantons. Such instantons are known for the $K = 1$ case [34, 35]. To estimate the action of such instantons we assume an infinitesimal ordering field that selects the ground state and consider then interaction energies as in (1). Assuming an instanton with width τ_0, the interaction term has the form $\eta(\Lambda\tau_0)^{2-2K}B_K$ while the mass term is $\sim M/\tau_0$, hence the action is minimized at $\Lambda\tau_0 \sim (\frac{M\Lambda}{(1-K)\eta B_K})^{\frac{1}{3-2K}}$ for $K < 1$; the numerical prefactor B_K is known at $K = 1$, $B_1 = \pi$. Note that the mass term and $K \neq 1$ set a finite scale for τ_0, unlike the $K = 1$ case. The instanton action is then

$$S_{inst} \approx M\Lambda \left(\frac{(2-2K)\eta B_K}{M\Lambda}\right)^{\frac{1}{3-2K}} \frac{3-2K}{2-2K} \quad (12)$$

Such instantons mean that the coordinate X_τ can tunnel between neighboring minima of the ordered $\langle\cos X_\tau\rangle$. Assuming independent instantons this would imply that $\langle(X_\tau - X_0)^2\rangle = D|\tau|$ has a finite diffusion constant. The

TABLE I. Correlations of the phases in Fig. 1 at $T = 0$.

correlation	delocalized	dissipative	periodic	localized
$\langle \cos X_\tau \rangle$	0	0	constant	1
$\langle \cos X_\tau \cos X_0 \rangle$	$\sim \lvert\tau\rvert^{-2K}$	$\sim \lvert\tau\rvert^{-(2-2K)}$	constant	1
$\langle (X_\tau - X_0)^2 \rangle$	$\sim \lvert\tau\rvert$	$\sim \ln\lvert\tau\rvert$		0

tunneling rate and D are then proportional to the instanton density, i.e. $D \sim \mathrm{e}^{-S_{inst}}$.

In particular we consider $K \to \frac{1}{2}$ and an instanton localized at $\tau = 0$. The dominant contribution for the instanton center at $|\tau| < \tau_0$ comes from $|\tau'| > \tau_0$ that involves $|X_\tau| \gg |X_{\tau'}|$ and $\int_{|\tau'|>\tau_0} |\tau'|^{-2K} \sim \frac{1}{2K-1}$ which diverges at $K \to \frac{1}{2}$, hence

$$S(K \to \tfrac{1}{2}) = \tfrac{1}{2} M \int_\tau (\dot{X})^2_\tau + \frac{\eta\Lambda(\Lambda\tau_0)^{1-2K}}{\pi(2K-1)} \int_\tau (1 - \cos X_\tau) + S' \quad (13)$$

S' comes from the instanton tails where $X_\tau, X_{\tau'}$ are small (up to 2π) and comparable. This action is similar to the well known sine-Gordon system, identifying $B_K \sim (2K-1)^{-1}$ whose instanton (or soliton) solution has a width $\tau_0 \sim (2K-1)^{\frac{1}{2}}$ and action $S_{inst} \sim (2K-1)^{-\frac{1}{2}}$. Assuming independent instantons the diffusion constant would diverges at $K = \frac{1}{2}$, i.e. $\ln D \sim (2K-1)^{-\frac{1}{2}}$. We propose that the whole range of the periodic phase in Fig. 1 has instanton solutions with a finite action, with an explicit solution provided by the sine-Gordon system at $K \to 1/2$. The tail contribution S' has a structure close to that of the 2nd term of Eq. (2), that may affect the interaction between instantons. Therefore, given the long range form of the interaction, to ascertain the correct behavior of $\langle (X_\tau - X_0)^2 \rangle$ at large time requires further study of how these instantons interact, which is left for the future.

We consider next the system at $K < 1/2$. This case has been studied in the context of discrete XY models [36–38] and was shown to have a phase transition in the limit that the coupling vanishes as a power of the system size, which in our case is $\beta = 1/T$, i.e. there is a critical value for $\eta(\beta)^{1-2K}$. Hence at $T = 0$ the system is fully ordered and $\cos X_\tau = 1$. Furthermore, instanton excitations would involve the effective coupling $\eta(\beta)^{1-2K}$, hence will have diverging action. Below a putative mean field transition temperature T_x where $4\eta M \Lambda T_x^{2K-1} = 1$ the system dynamics would be similar to that of the periodic phase.

Let us conclude this part by noting that the hypothesis made at the beginning to neglect $X_\tau - X_0$ compared to τ is indeed justified in all the phases. Furthermore, note that although the results of the present paper are derived in the large-N limit, we of course expect them to extend to a finite number of component as well. E.g. for the Coulomb box case deviations due to finite N appear at exponentially small temperatures [39].

Finally we discuss possible realizations of our model with various collective coordinates that are potential candidates for experimental studies. A first possibility to obtain (1) is to use a fermion Coulomb box [40]. Following the Ambegaokar-Eckern-Schön mapping [41] one introduces a phase X_τ such that $\dot{X}_\tau$ measures the charge in the box while the charging energy corresponds to $1/M$. The kernel in (1) is then $\sum_\alpha G_{\alpha,i}(\tau - \tau') \sum_k G_{k,i}(\tau' - \tau)$ where i is the channel index, α, k are internal quantum numbers of the dot and LL, respectively, and the Green's functions are for either free fermions on the dot, $\sim 1/(\tau - \tau')$ while for fermions in the LL it is $\sim |\tau - \tau'|^{-\frac{1}{2}(K_f + 1/K_f)}$. Hence an effective action of the form (1) with $2K = 1 + \frac{1}{2}(K_f + 1/K_f)$, realizing only $K > 1$ cases. In case that the LL terminate at the Coulomb box a boundary Green's function [15] is needed $G_{x=0,i}(\tau - \tau') \sim |\tau - \tau'|^{-1/K_f}$, hence $2K = 1 + \frac{1}{K_f}$ and $K < 1$ is realizable. A second possible realization corresponds to a BEC with a phase θ_τ that weakly couples to bosonic LL's with boson operators $\Psi_n(\tau)$ as $g\mathrm{e}^{i\theta_\tau}\Psi_n(\tau) + h.c.$. The average involves now the boson's Greens function $\sim |\tau - \tau'|^{1/2K_b}$, $K_b \to \infty$ for noninteracting bosons and K_b decreases to 1 for on site repulsion $U \to \infty$. Hence (1) is realized with $K = 1/4K_b$ and $K < 1$ is possible. In analogy with BEC, a superconducting grain can Josephson couple to fermionic LL's providing a third type of realization; here again $K < 1$ can be realized.

In conclusion, we have studied the physics of LL environments that couple to a collective coordinate such as a impurity position, charge of a Coulomb box, a phase of a BEC or that of a superconducting grain. We have shown that the coupling to the bath leads to various phases for the collective coordinate ranging from delocalized, dissipative, periodic and localized. Our results are summarized in Fig. 1 and Table I, showing the distinctions among the various phases. We believe that the large set of realizations for the collective coordinate and the various phase transitions will stimulate further research.

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